



# A simple model for predicting ice formation timing, thickness, and quality in lakes

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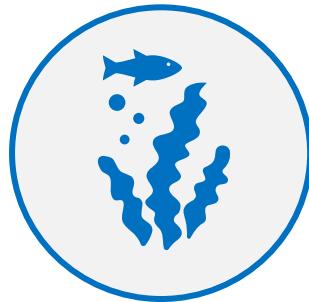
<https://www.linkedin.com/in/sebastianopiccolroaz/>

# Motivation

Why is lake ice important?



Safety reasons  
(transportation)



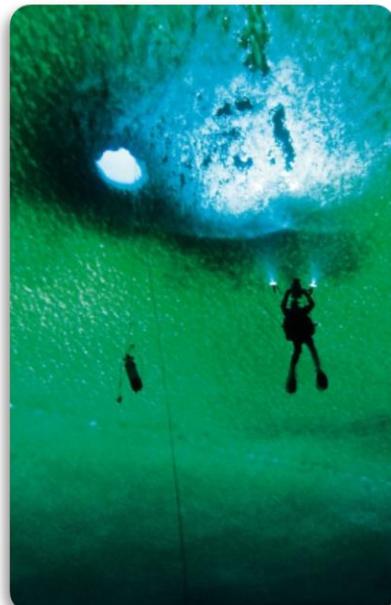
Impact on aquatic  
ecosystems  
(light penetration, turbulence)



Human activities  
(fishing, skating, culture)  
Knoll et al. (2019)



Climate change  
(relevant indicator)

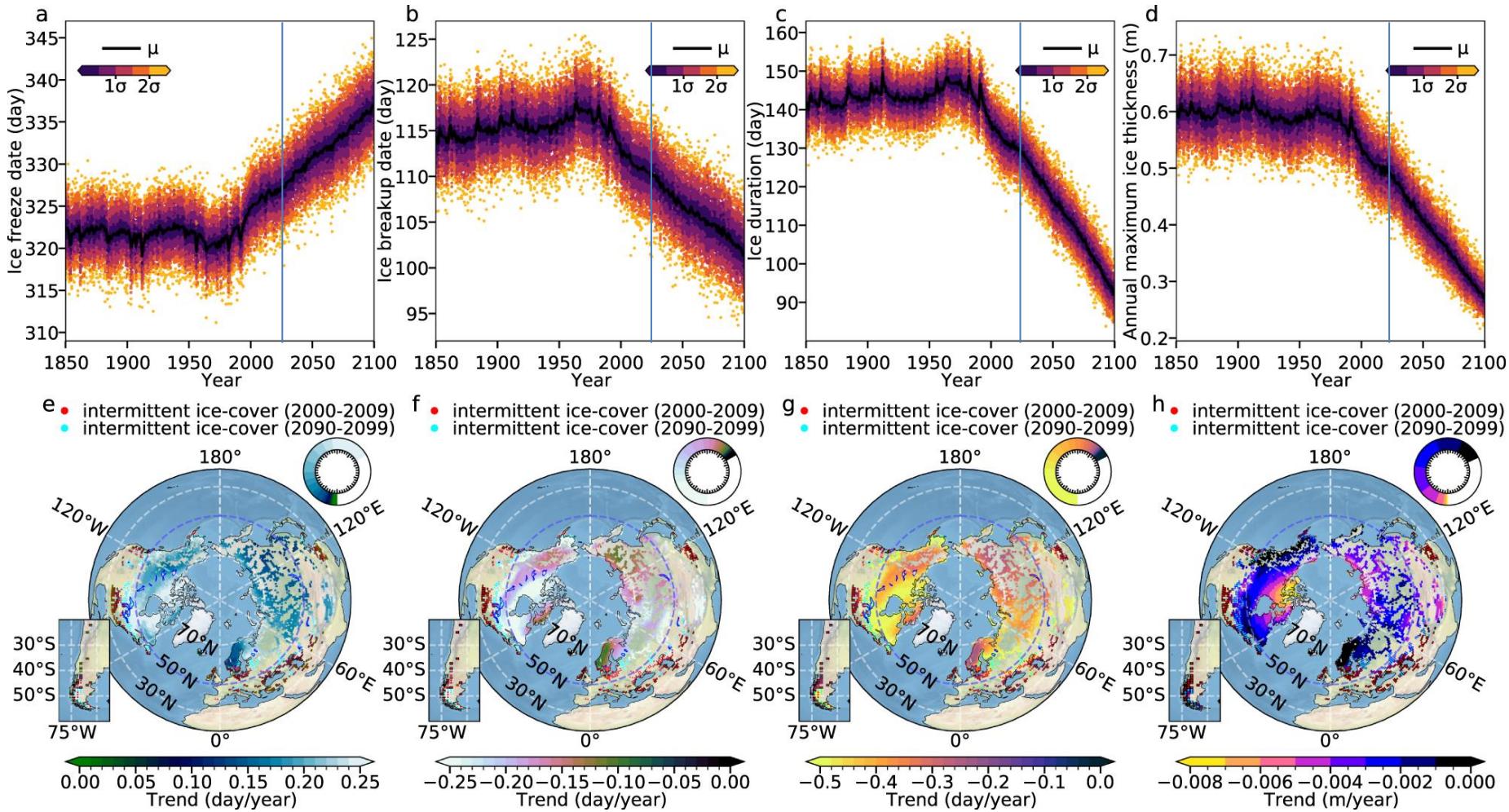


# Motivation

In the next 80 years

Ice coverage: **- 38 days**

Maximum ice thickness: **- 0.23 m**



# The *air2water* model

**Hybrid** physically-based/statistical model for **LST** prediction

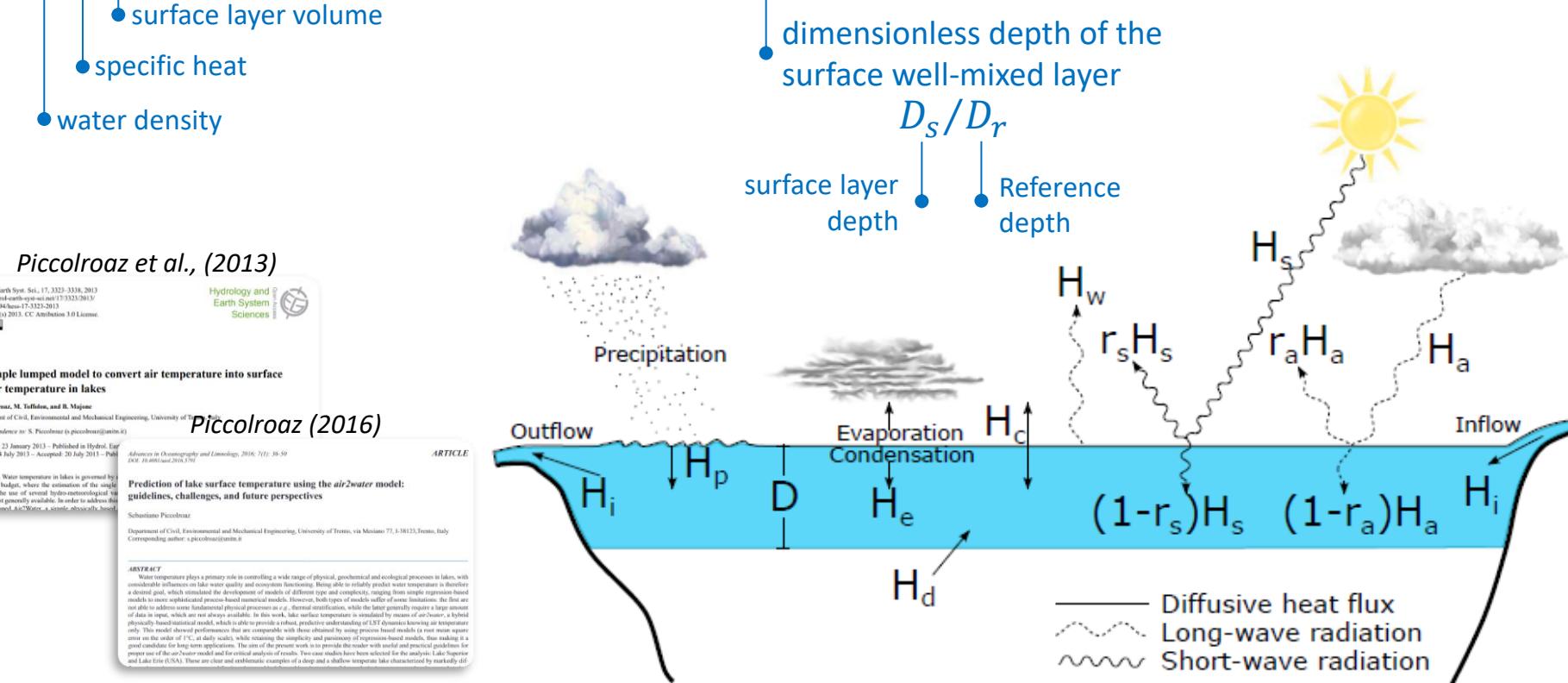
Relies on **few input data** (only air temperature), while retaining the same **high performance** of deterministic models

$$\rho_w c_p V_s \frac{dT_w}{dt} = \phi_{net} A$$

net heat flux  
surface area

$\rightarrow$

$$\frac{dT_w}{dt} = \frac{1}{\delta} \left\{ a_1 + a_2 T_a - a_3 T_w + a_5 \cos \left[ 2\pi \left( \frac{t}{t_y} - a_6 \right) \right] \right\}$$



# The *air2water* model

**Hybrid** physically-based/statistical model for **L SWT** prediction

Relies on **few input data** (only air temperature), while retaining the same **high performance** of deterministic models

$$\rho_w c_p V_s \frac{dT_w}{dt} = \phi_{net} A$$

Legend:

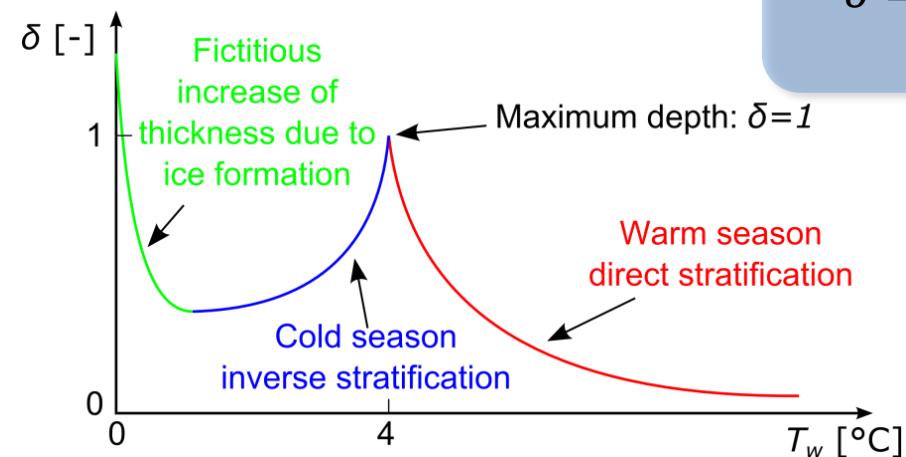
- net heat flux
- surface area
- surface layer volume
- specific heat
- water density

$$\frac{dT_w}{dt} = \frac{1}{\delta} \left\{ a_1 + a_2 T_a - a_3 T_w + a_5 \cos \left[ 2\pi \left( \frac{t}{t_y} - a_6 \right) \right] \right\}$$

dimensionless depth of the surface well-mixed layer

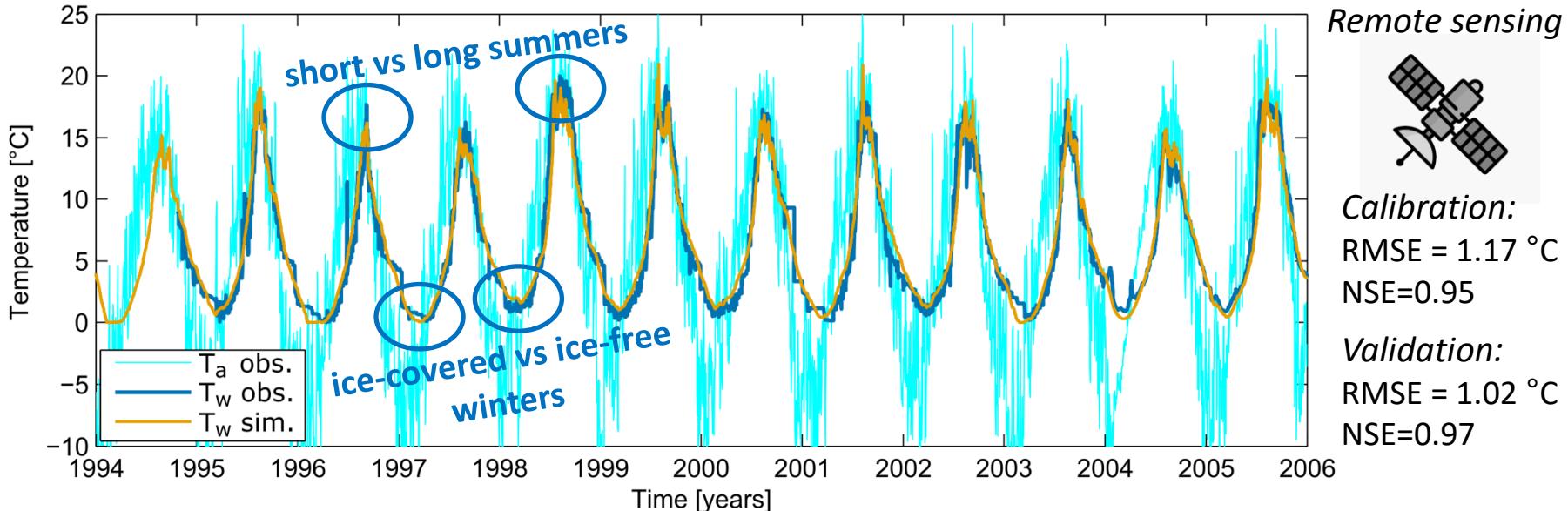
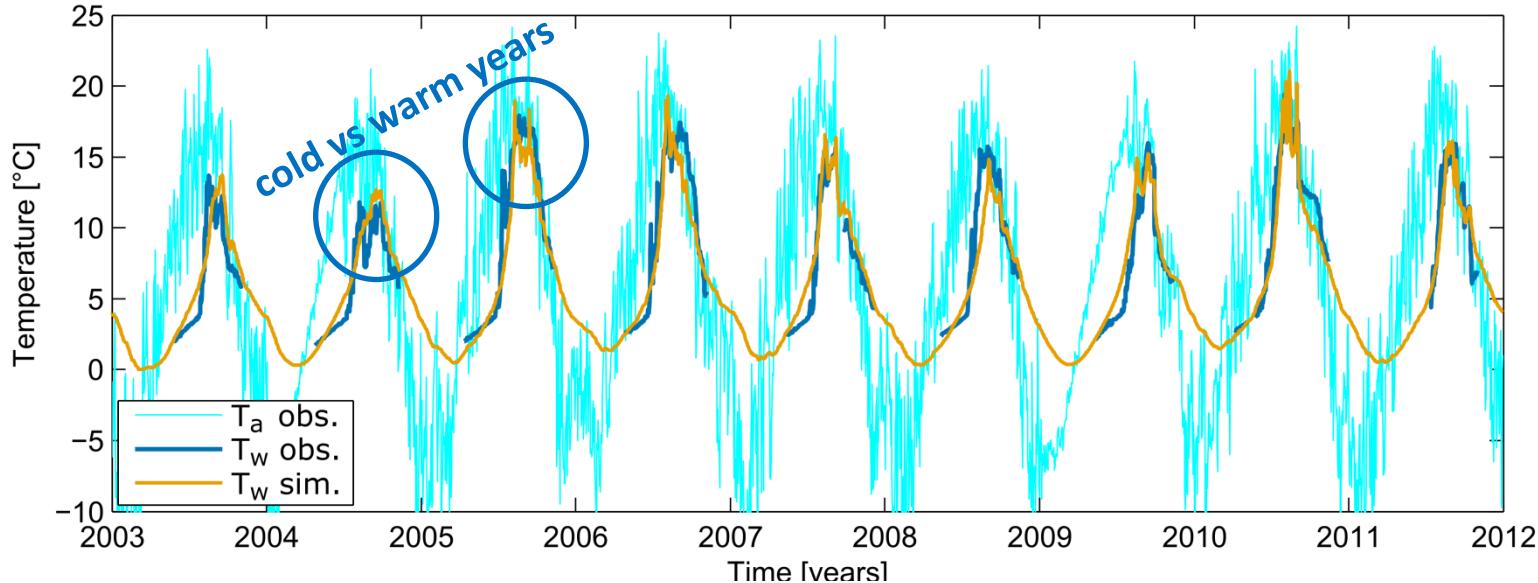
$$\delta = \exp \left( -\frac{T_w - T_h}{a_4} \right) \quad \text{for } T_w \geq T_h$$

$$\delta = \exp \left( -\frac{T_h - T_w}{a_7} \right) + \exp \left( -\frac{T_w}{a_8} \right) \quad \text{for } T_w < T_h$$



A simple **ODE** with an **empirical closure** for stratification  
8 parameters

# Application to Lake Superior



# The ice module

Weyhenmeyer et al. (2022)



**Black ice**

Formed from the lake  
water freezing



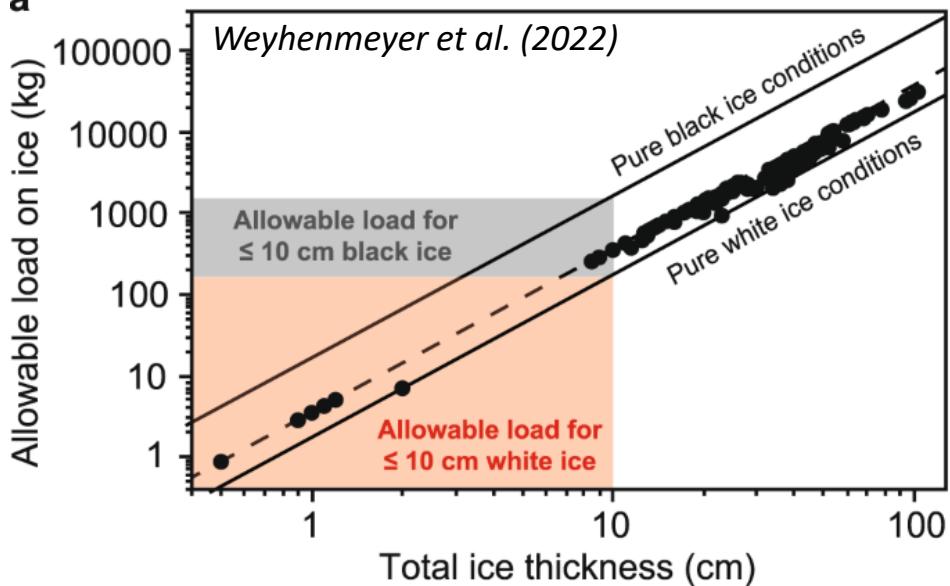
**White ice**



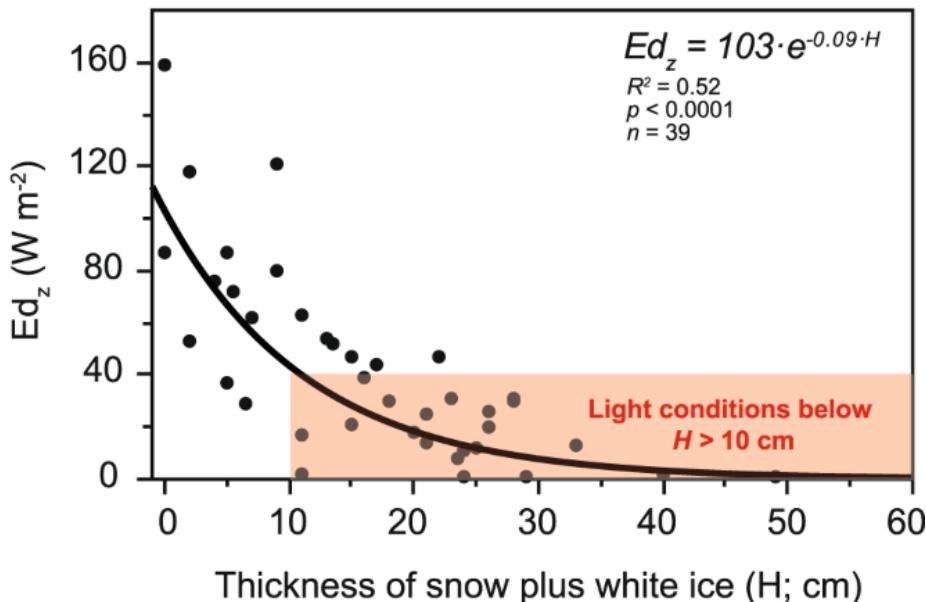
Formed when snow  
accumulates on ice, melts, and  
refreezes. Or when rain falls on  
the snow layer or the snow  
layer is flooded.

# The ice module

a



b



## Safety

White ice is unstable with a low bearing capacity than black ice



## Under-ice ecology

White (*opaque*) ice has higher reflectance than black (*clear*) ice, reducing the amount of sunlight penetrating through ice.

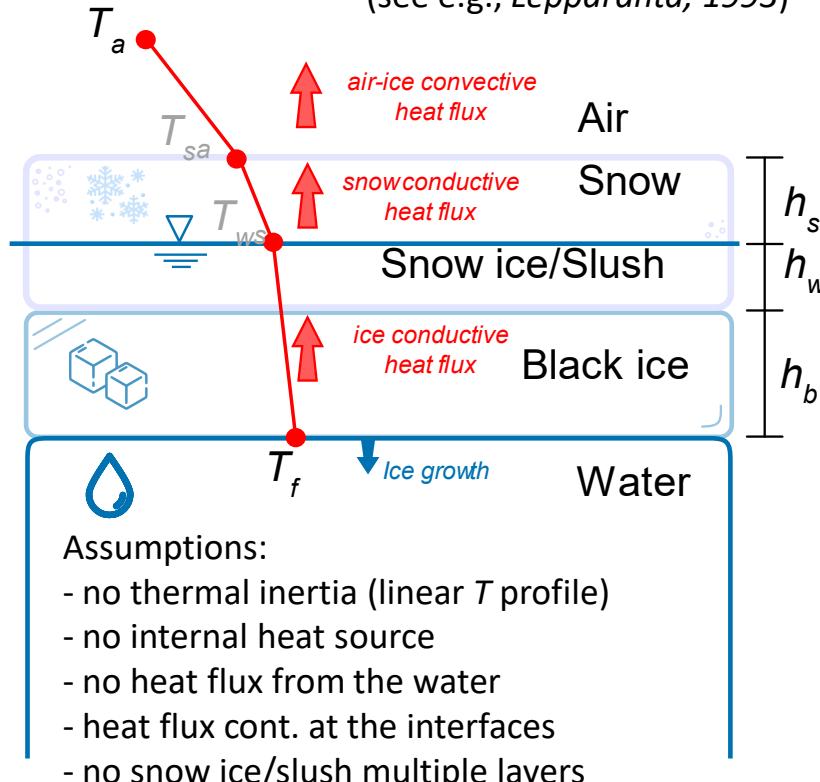


# The ice module

## Growth equation

$$T_a < T_f + a_9$$

Stefan's Law including  
ice-snow-atmosphere coupling  
(see e.g., Leppäranta, 1993)

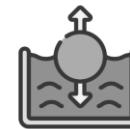


$$\frac{dh_b}{dt} = \frac{1}{L_f \rho_i h_b / k_i + h_w / k_i + h_s / k_s + 1/a_{10}} (T_f - T_a + a_9)$$

Known thermophysical  
properties of ice



<https://www.klikk.no/produkthjemmesider/villmarksliv/viltogvariert/stalis-3136455>



Algorithm for flooding  
(buoyancy)



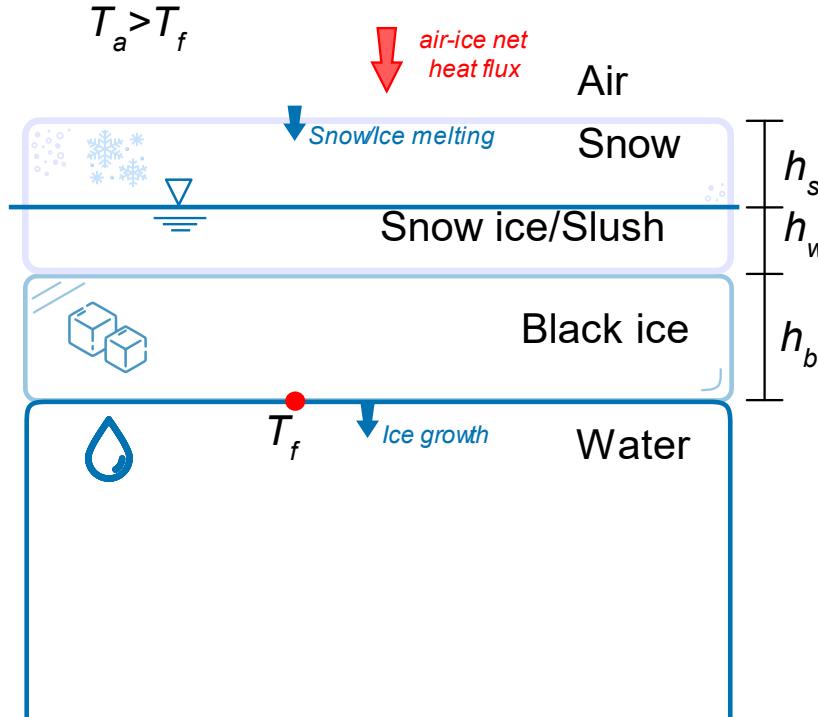
Observed/modelled precipitation  
(liquid and solid)

$a_9$ : to account for possible bias  
correction of the (only) forcing  $T_a$   
 $a_{10}$ : air convective heat transfer coefficient

# The ice module

**Melting equation** Surface melting is triggered

$T_a > T_f + a_9$  whenever the surface temperature is at the **melting point**



$$\frac{dh_l}{dt} = -\frac{\phi_{net}}{L_f \rho_l}$$

The net surface heat flux determines the rate of melting.  
( $l$  = upper layer)



$$\rho_w c_p V_s \frac{dT_w}{dt} = \phi_{net} A$$

$$\frac{dT_w}{dt} = \frac{\phi_{net} A}{\rho_w c_p V_s} = \frac{\phi_{net}}{\rho_w c_p D_s}$$

$$= \frac{1}{\delta} \left\{ a_1 + a_2 T_a - a_3 T_w + a_5 \cos \left[ 2\pi \left( \frac{t}{t_y} - a_6 \right) \right] \right\}$$

$$\delta = D_s / D_r$$

$$\phi_{net} = D_r \rho_w c_p \left\{ a_1 + a_2 T_a - a_3 T_w + a_5 \cos \left[ 2\pi \left( \frac{t}{t_y} - a_6 \right) \right] \right\}$$

$$T_w = T_f = 0^\circ C$$

$$\frac{dh_l}{dt} = -\frac{a_{11} D_r \rho_w c_p}{L_f \rho_l} \left\{ a_1 + a_2 T_a + a_5 \cos \left[ 2\pi \left( \frac{t}{t_y} - a_6 \right) \right] \right\}$$

Known thermophysical properties of water/ice

$a_{11}$ : a parameter mainly accounting for possible differences between  $\phi_{net}$  parameterization during ice-free/ice-on seasons (see e.g., albedo)

# The complete model

A system of two independent ODEs, with a total of 10 parameters:

$$\frac{dT_w}{dt} = \frac{1}{\delta} \left\{ a_1 + a_2 T_a - a_3 T_w + a_5 \cos \left[ 2\pi \left( \frac{t}{t_y} - a_6 \right) \right] \right\}$$

Growth

$$\frac{dh_b}{dt} = \frac{1}{L_f \rho_i} \frac{T_f - T_a - a_9}{h_b/k_i + h_w/k_i + h_s/k_s + 1/a_{10}}$$

Melting

$$\frac{dh_l}{dt} = - \frac{a_{11} D_r \rho_w c_p}{L_f \rho_l} \left\{ a_1 + a_2 T_a + a_5 \cos \left[ 2\pi \left( \frac{t}{t_y} - a_6 \right) \right] \right\}$$



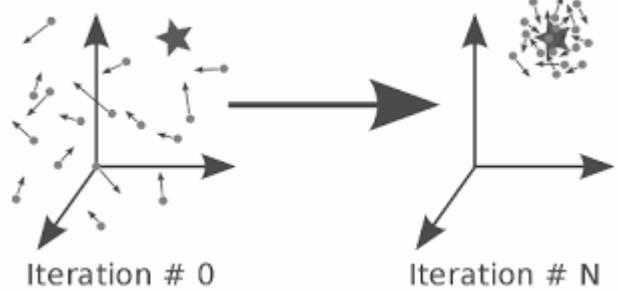
Model calibration and performance:

Multi-objective optimization problem:

$$NSE_{tot} = \beta NSE_{LSWT} + (1 - \beta) NSE_{Ice}$$

$$NSE = 1 - \frac{\sum_{i=1}^n (obs_i - sim_i)^2}{\sum_{i=1}^n (obs_i - \overline{obs})^2}$$

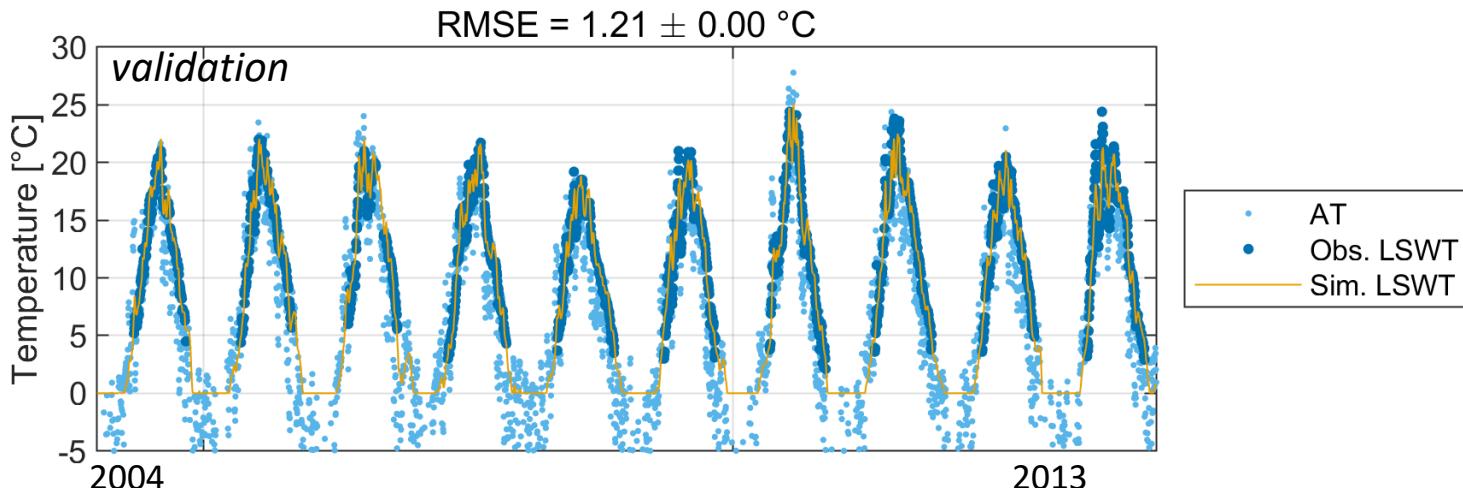
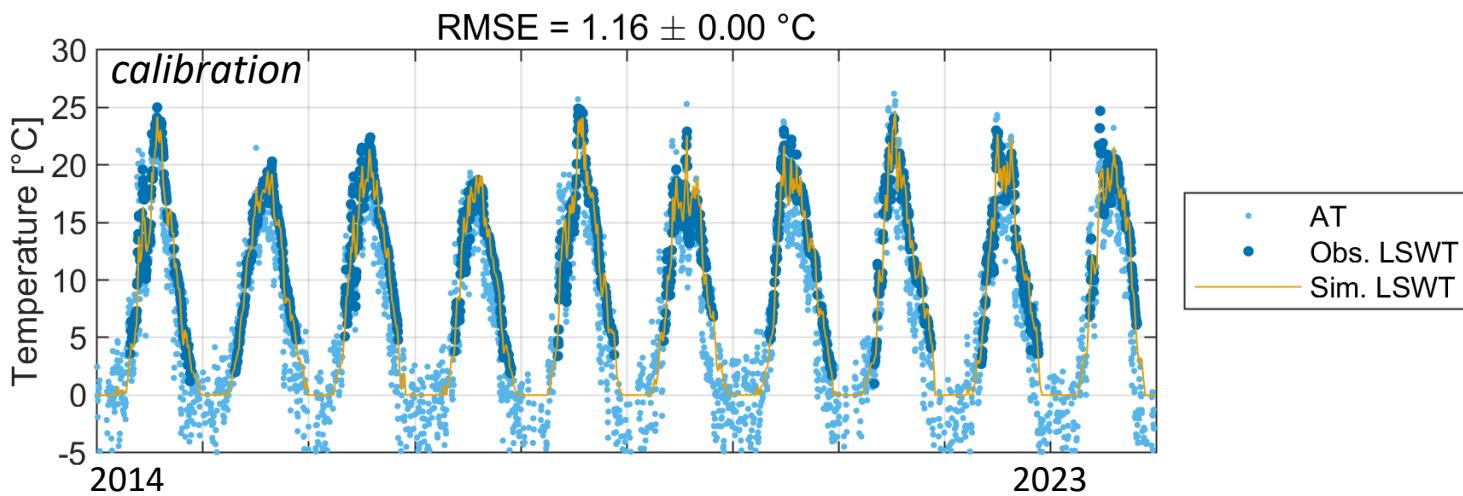
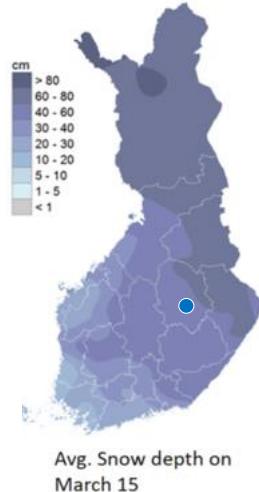
PSO (Particle Swarm Optimization)



# Results | Lake Kallavesi

- Boreal oligotrophic lake
- Central Finland
- Mean depth 8.9 m

$NSE_{tot} = NSE_{LSWT}$   
*air2water* model with 6 parameters

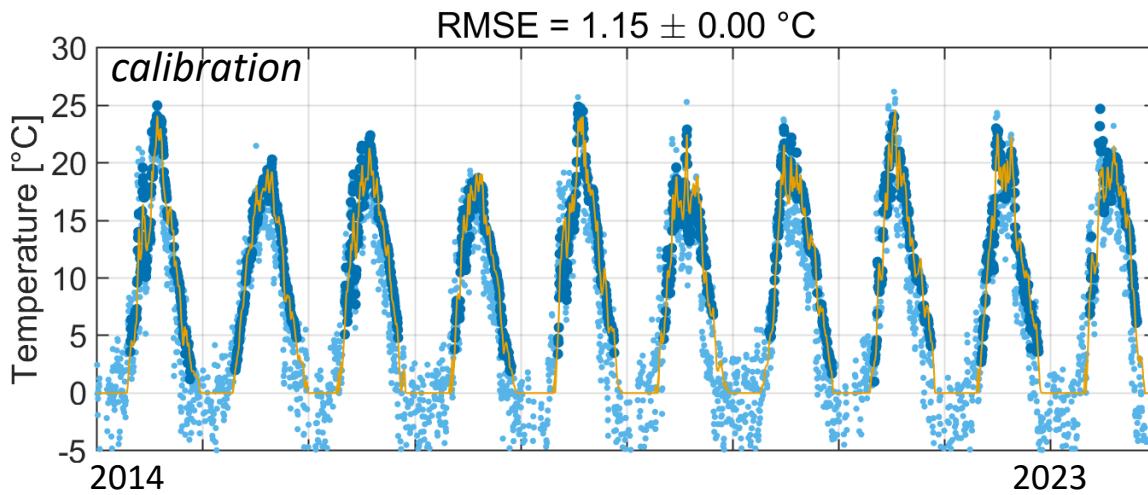
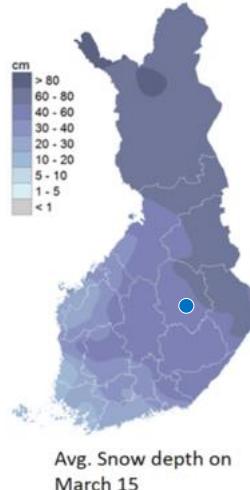


# Results | Lake Kallavesi

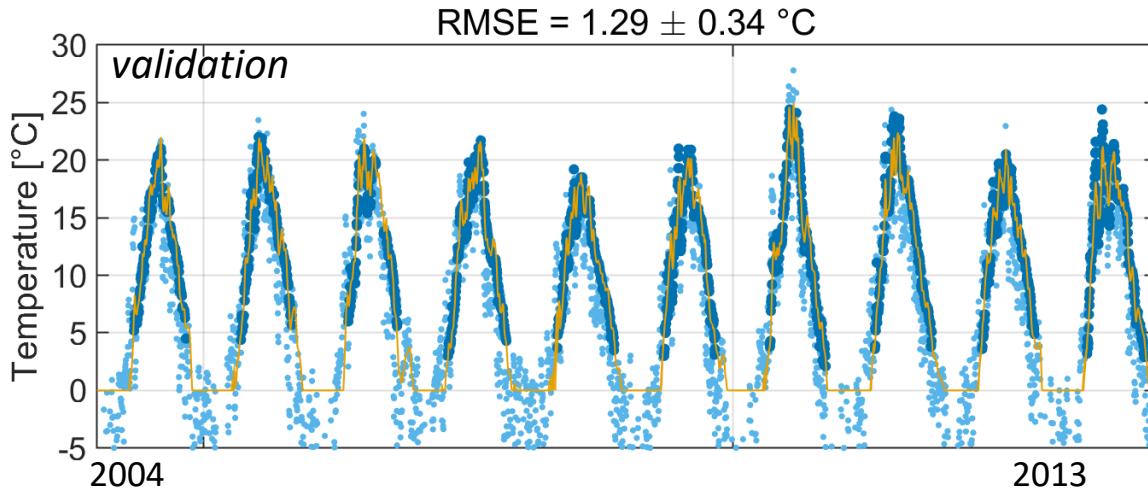
- Boreal oligotrophic lake
- Central Finland
- Mean depth 8.9 m

$$NSE_{tot} = \beta NSE_{LSWT} + (1 - \beta) NSE_{Ice}$$

Case when  $\beta = 1$



Overall similar performance.  
Slight worsening in validation but ...



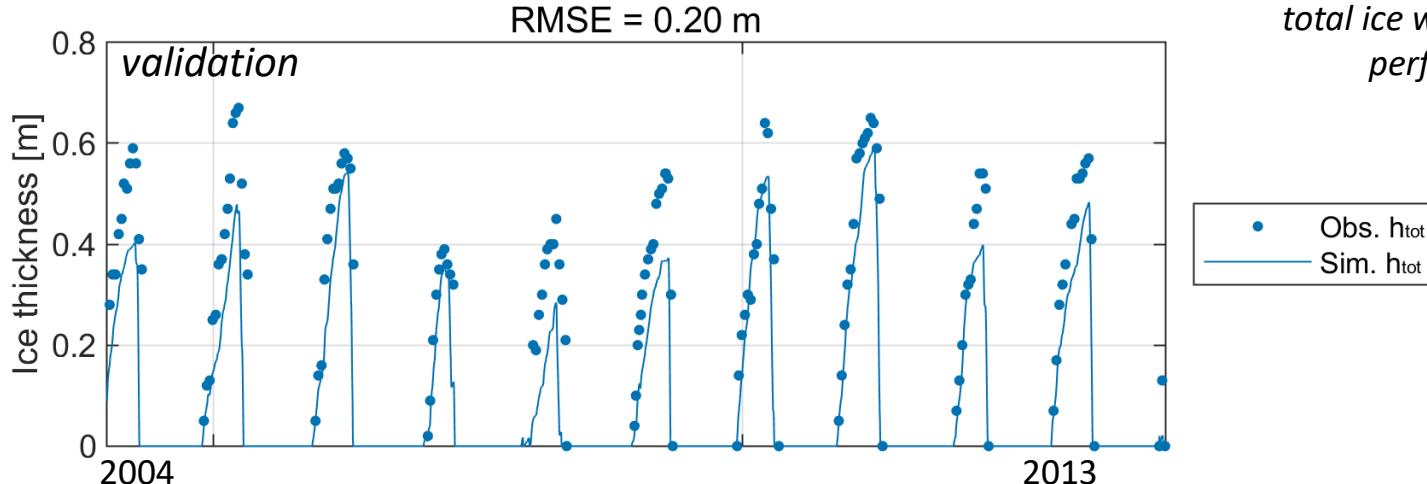
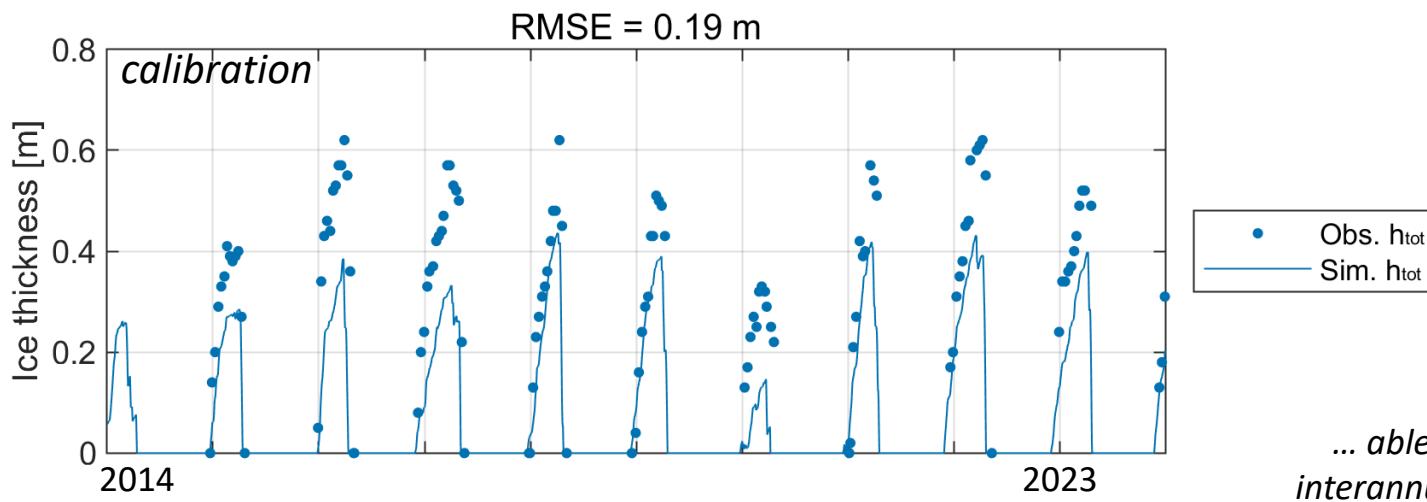
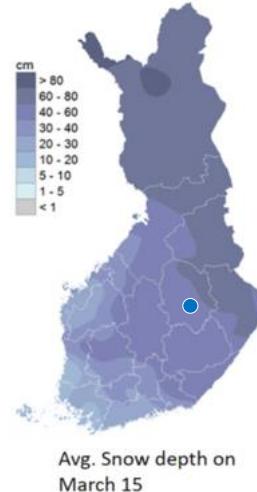
air2water

# Results | Lake Kallavesi

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$$NSE_{tot} = \beta NSE_{LSWT} + (1 - \beta) NSE_{Ice}$$

Case when  $\beta = 1$



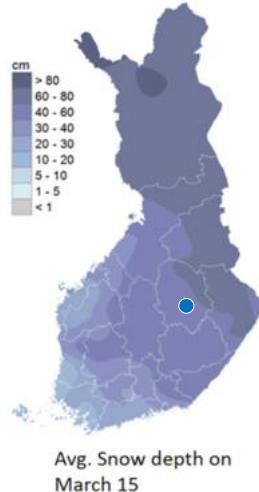
... able to reproduce  
interannual variability of  
total ice with a reasonable  
performance ...

# Results | Lake Kallavesi

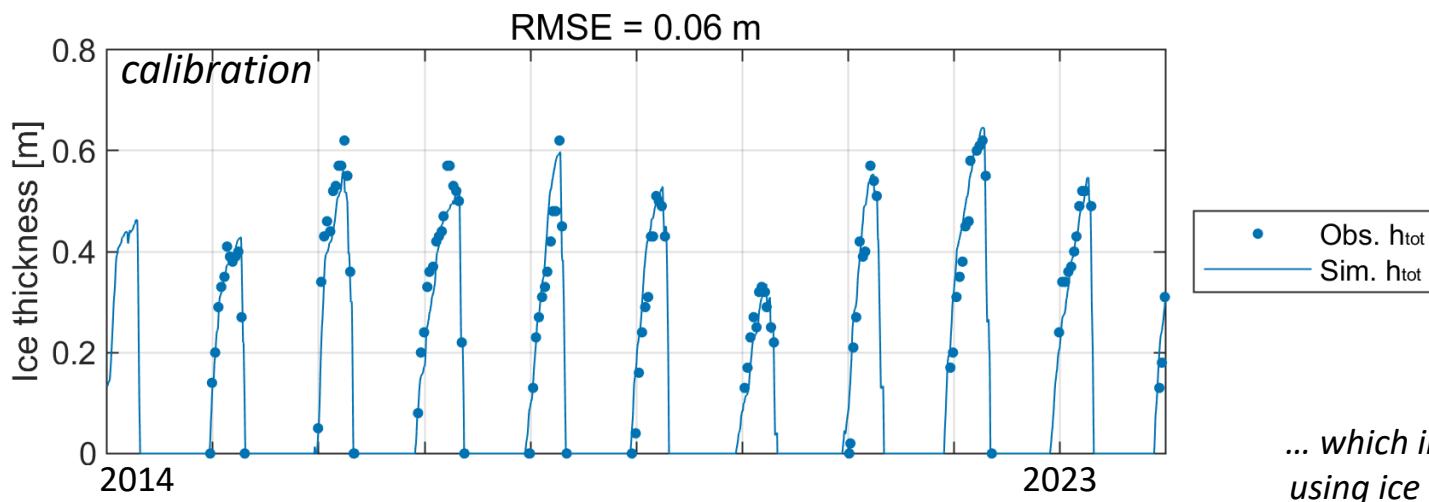
- Boreal oligotrophic lake
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$$NSE_{tot} = \beta NSE_{LSWT} + (1 - \beta) NSE_{Ice}$$

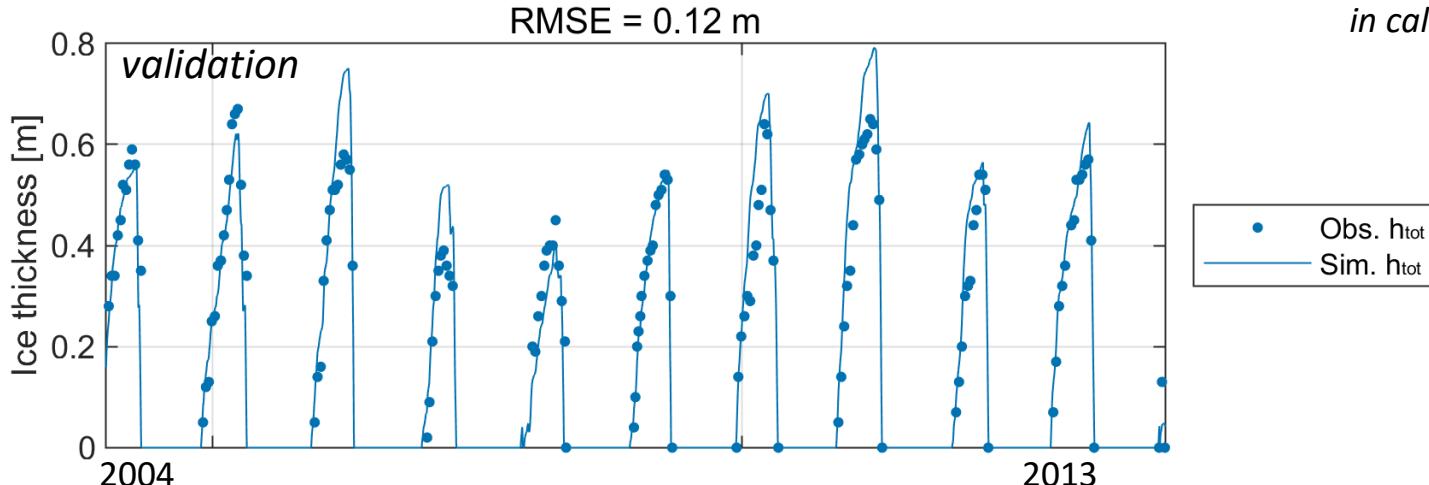
Case when  $\beta = 0.5$



Neuenschwander et al. (2020)



... which increases when  
using ice thickness data  
in calibration ...

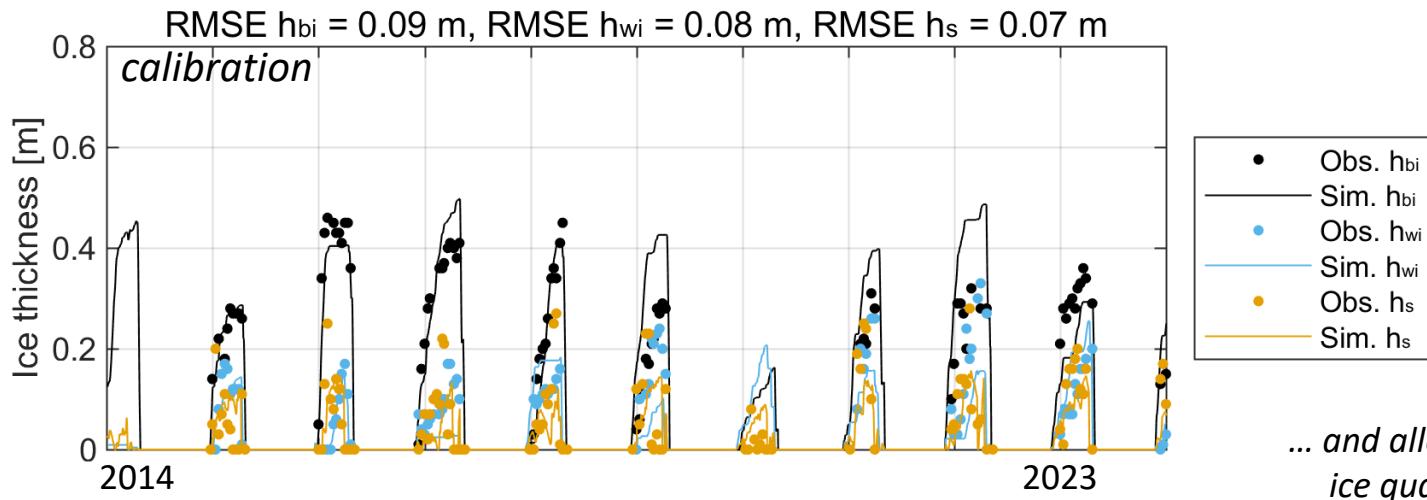
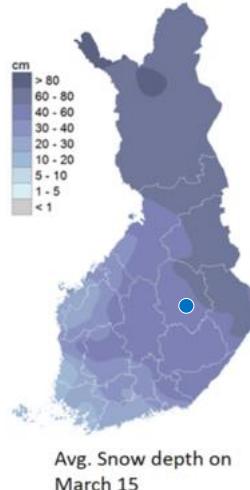


# Results | Lake Kallavesi

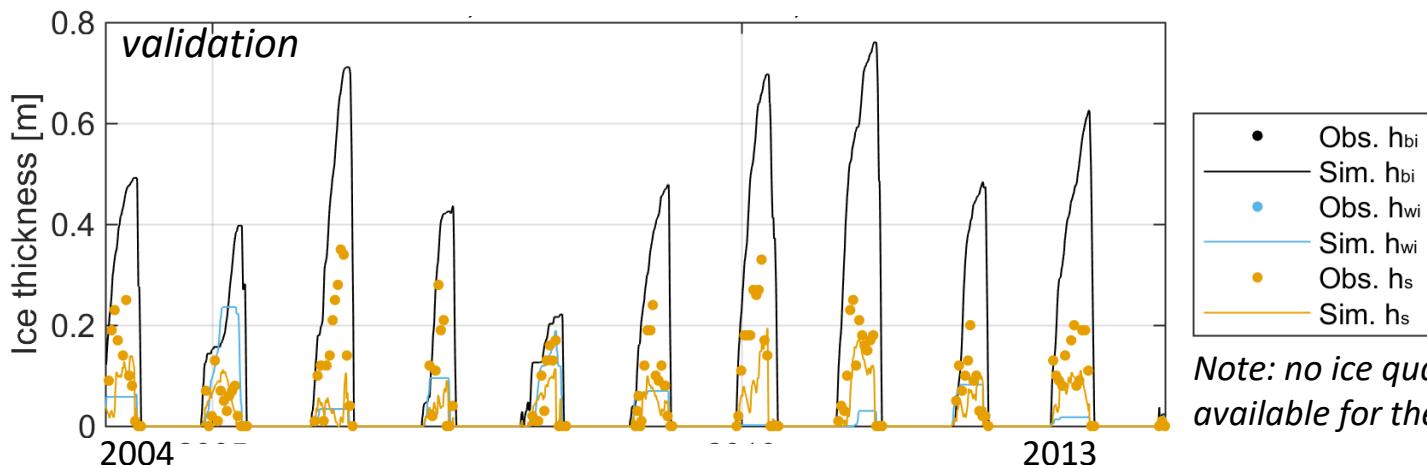
- Boreal oligotrophic lake
- Central Finland
- Mean depth 8.9 m

$$NSE_{tot} = \beta NSE_{LSWT} + (1 - \beta) NSE_{Ice}$$

Case when  $\beta = 0.5$

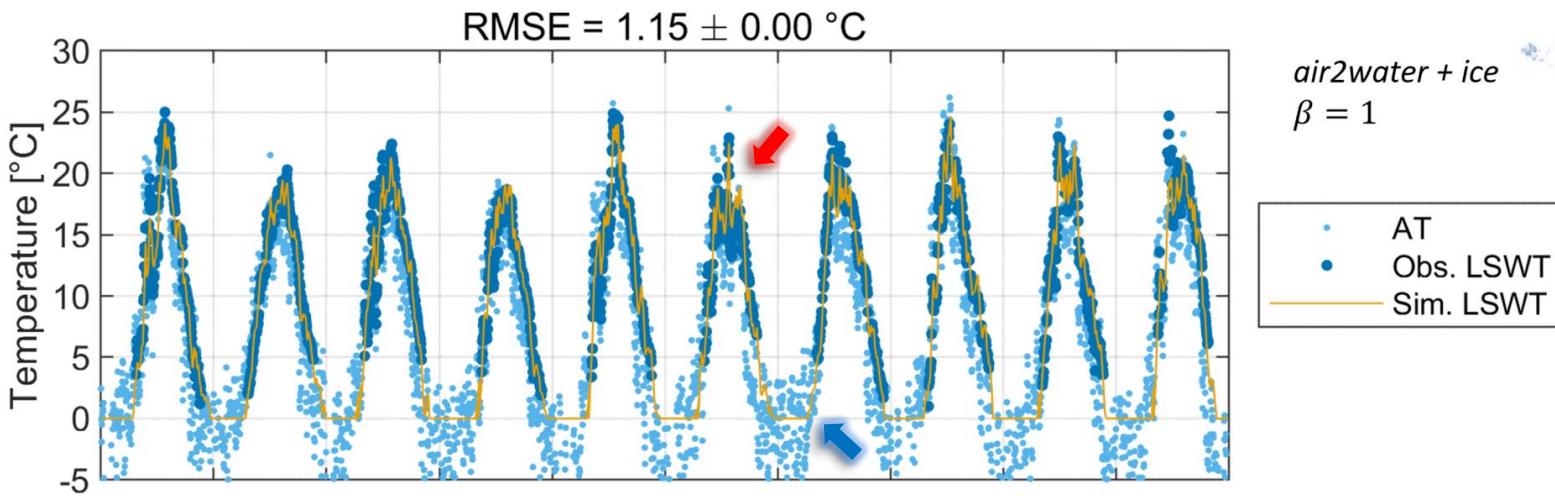


... and allows predicting  
ice quality as well.



# Results | Lake Kallavesi

- Boreal oligotrophic lake
- Central Finland
- Mean depth 8.9 m

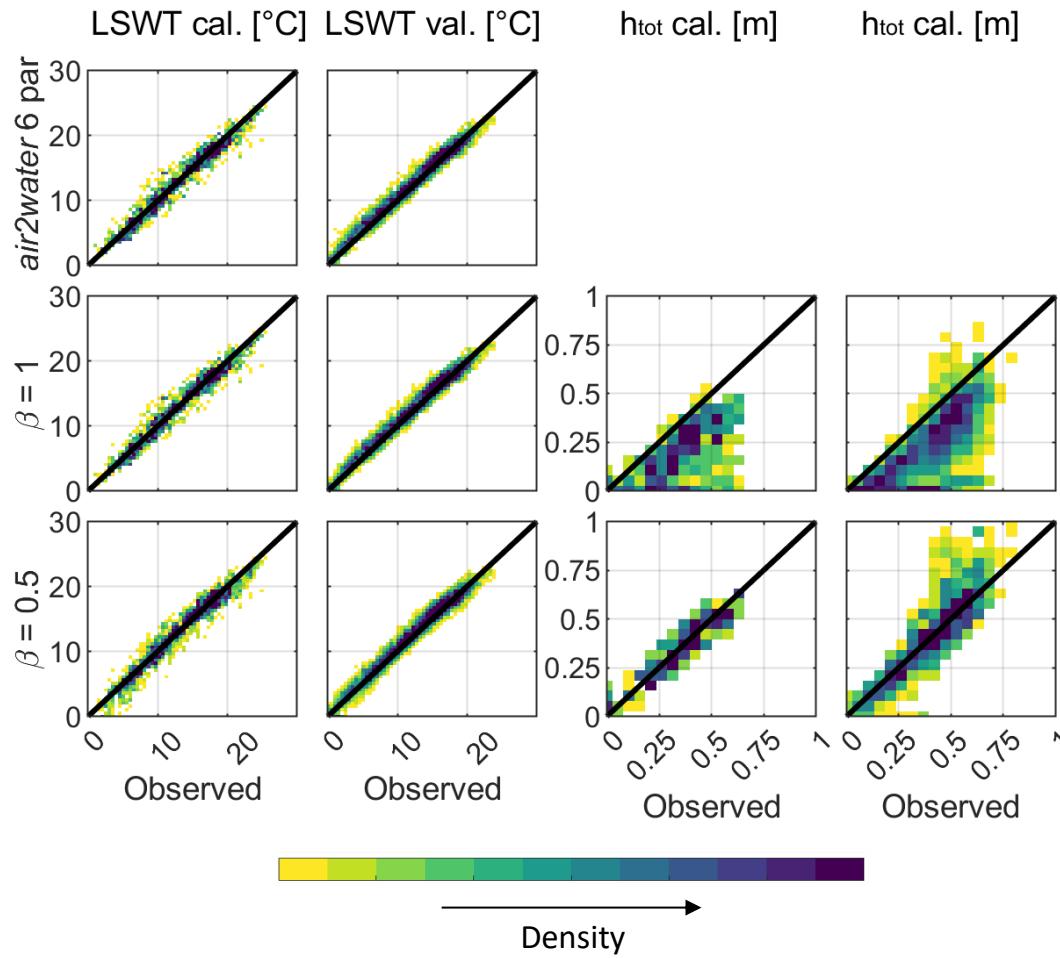
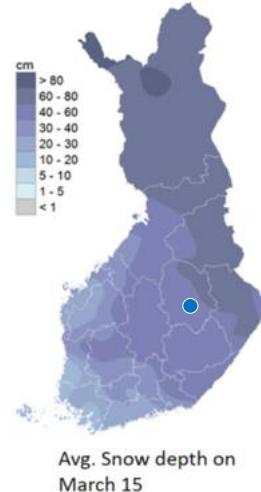


*There is no free lunch:*

- slight worsening in LSWT prediction
- mainly in the summer period (largest natural variability)
- but better capturing of the ice on-off timing

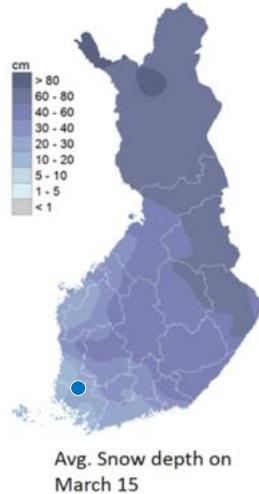
# Results | Lake Kallavesi

- Boreal oligotrophic lake
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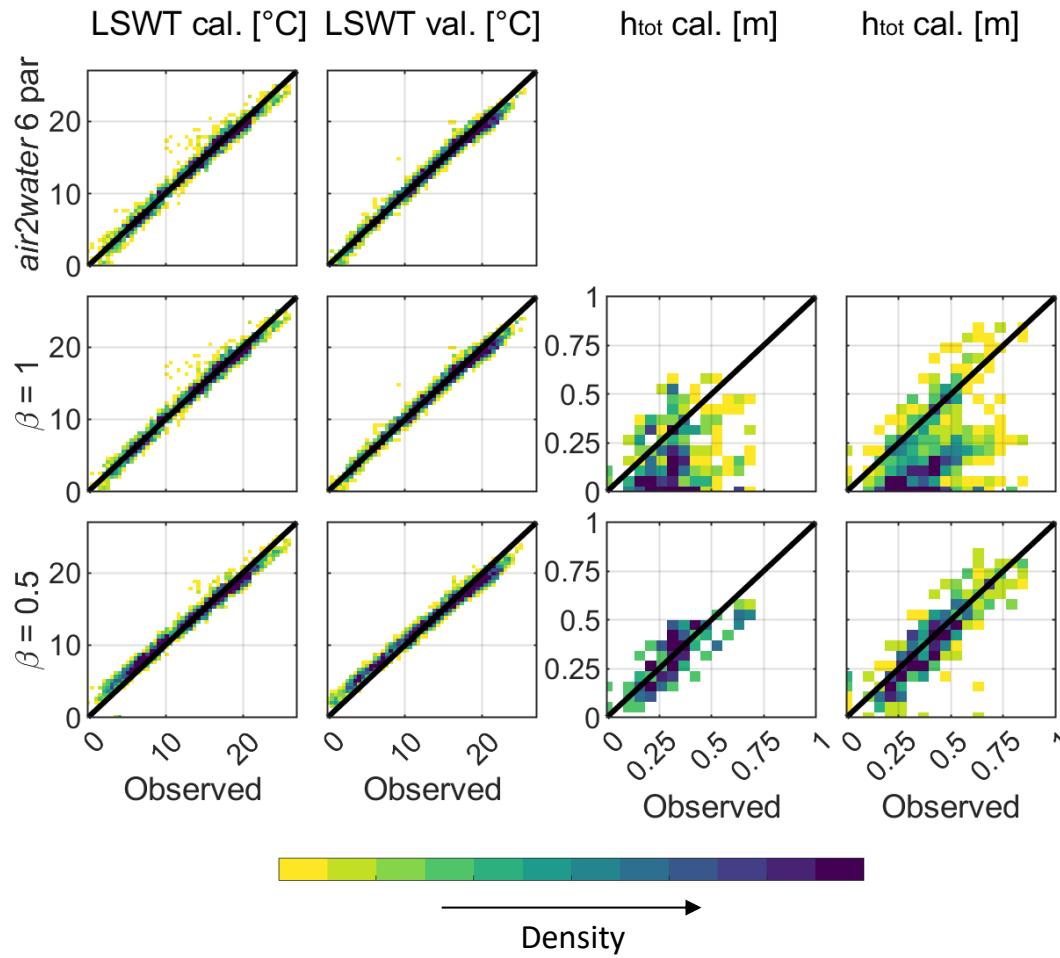


# Results | Lake Pyhajarvi

- Eutrophic lake
- Southern Finland
- Mean depth 5.4 m

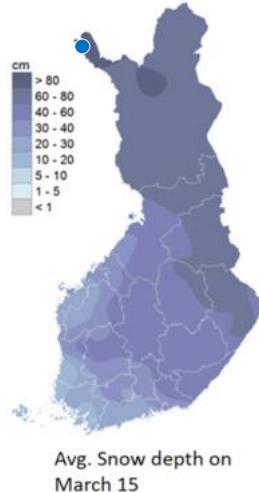


Neuenschwander et al. (2020)

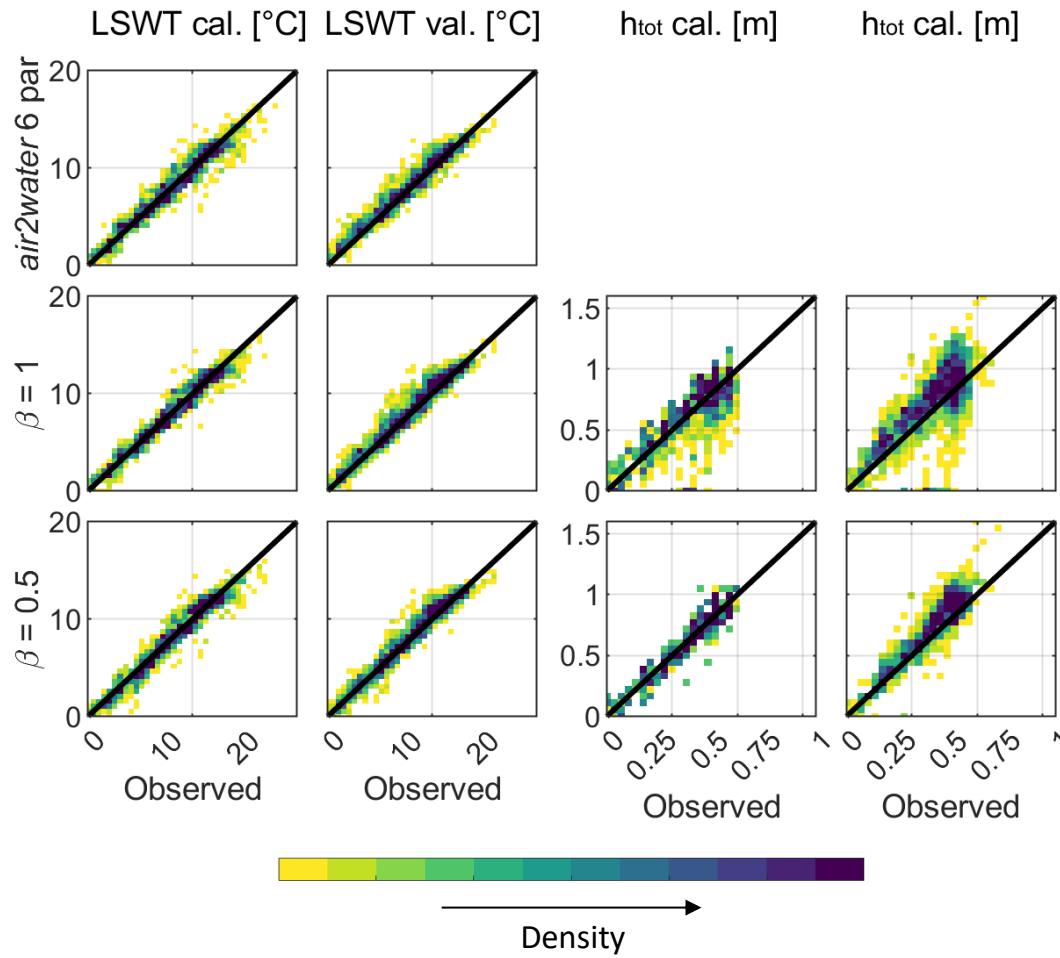


# Results | Lake Kilpisjarvi

- Oligotrophic lake
- Northern Finland (tundra)
- Mean depth 19.5 m

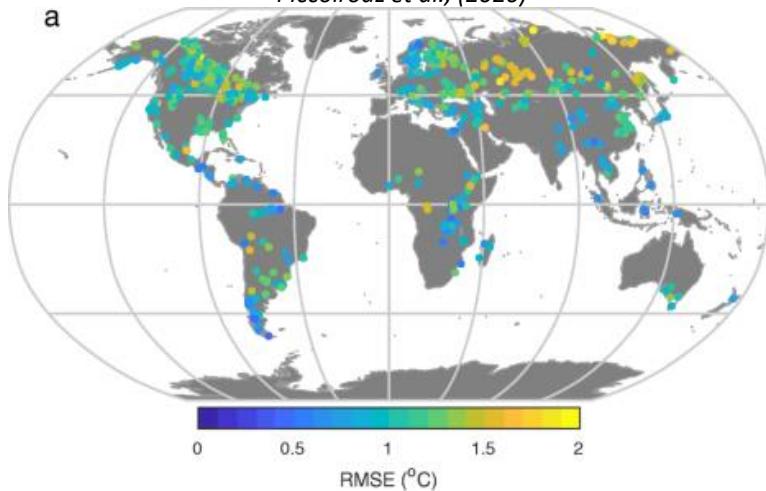


Neuenschwander et al. (2020)

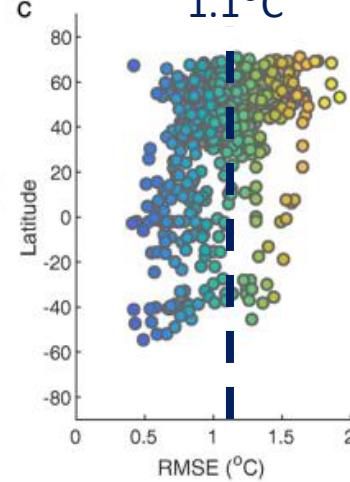


# Current developments

Piccolroaz et al., (2020)



1.1°C

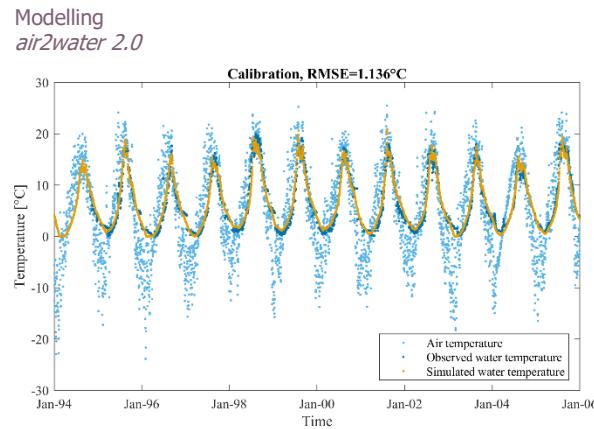


Regionalization of the model parameters



- application to non monitored lakes
- coupling with climate models

HydroSQUAS  
sites.google.com/unirn.it/hydrosquas/home/web-browser/Modelling



Download

Powered by:  
  
Google Earth Engine  
  
  
a research toolkit for Particle Swarm Optimization in Python

Release of web portal for running the model on-line



# Conclusions

We extended a **simple model** (*air2water*) adding an ice module to predict total **ice thickness**, using **air temperature** as the only input forcing.

We added **precipitation** as input forcing and modified the ice module in order to simulate **ice quality** (black and white ice, but also slush and snow thicknesses)

**Good model performance**, coherent with that of more complex models:

$$RMSE_{LSWT} \sim 1^{\circ}C$$

$$RMSE_{Ice} \sim 0.1 m$$

When using **only LSWT observations** for model calibration, the ice thickness model **performance decreases** (but still  $RMSE_{Ice} \sim 0.1 m$ ).

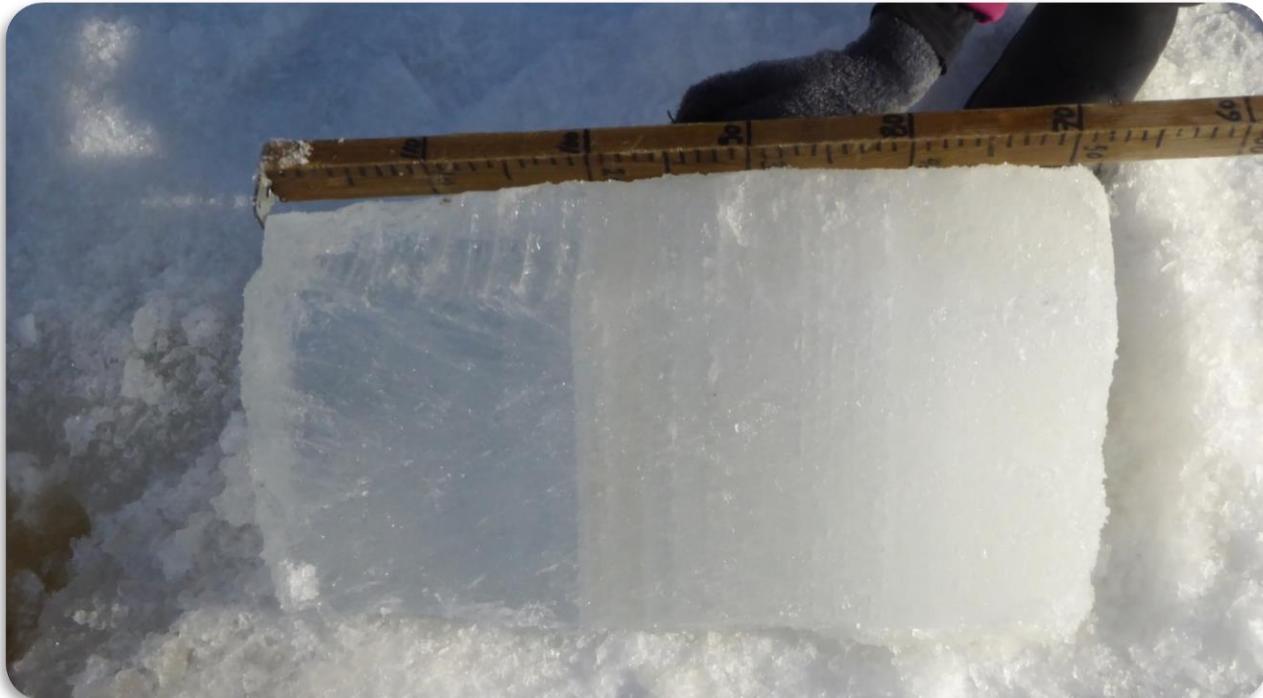
In these cases, the performance of ice quality simulation worsens but is still reasonable. Access to good (representative) precipitation data is needed.

The model is **simple** and **parsimonious** but **physically based** and looks robust. It can be a valid tool for large-scale and long-term simulations.



# A simple model for predicting ice formation timing, thickness, and quality in lakes

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## Special Collection Call for Submissions

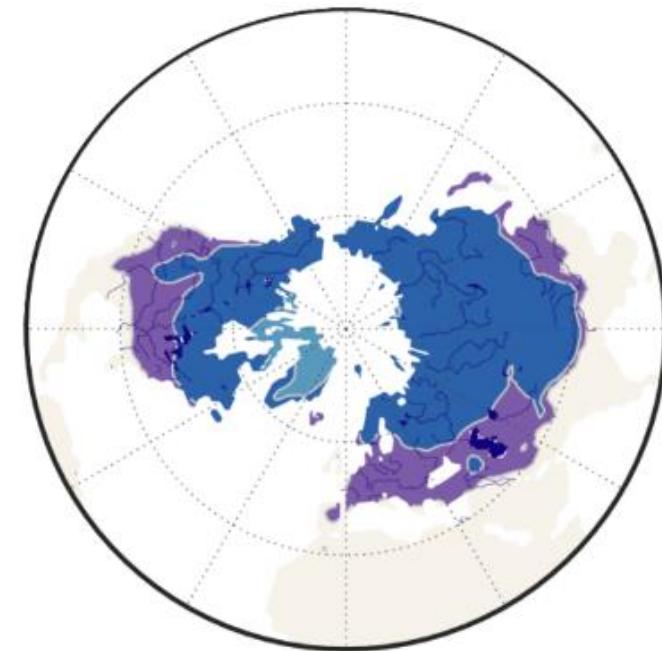
Integrating In Situ, Remote Sensing, And Physically Based Modeling Approaches to Understand Global Freshwater Ice Dynamics

Open for Submissions:

**1 April 2023**

Submission Deadline:

**31 December 2024**



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**Sebastiano Piccolroaz**, University of Trento, Italy

**Joshua Culpepper**, York University, Canada

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